Geometric mean and harmonic mean

GEOMETRIC MEAN

GM = Geometric mean of a set of an n observations is the nth root of their product .
$GM = (x_1 . x_2 . x_3 . . . . . x_n)^{1/n}$
Where, n is the total no of observations.
$X_1, x_2, ……..x_n.$ these are your observations.

Q1) Find the GM of 2 and 8. (x_1, x_2)
N? = 2

Ans. $GM = (2 \times 8)^{½}$
$= (16)^{1/2} = (2x2x2x2)^{1/2} \Rightarrow (4 \times 4)^{1/2} \Rightarrow [(4)^2]^{1/2}$
GM = 4.

Q2) Find the GM of 2, 9, 12.
$GM = (2 \times 9 \times 12)^{1/3}$
$= (2 \times 3 \times 3 \times 2 \times 2 \times 3)^{1/3}$
$= (2^3 \times 3^3)^{1/3}$ [taking a group of threes. Three 2s and three 3s]
$\Rightarrow 2 \times 3$
GM = 6

Q3) Find the GM of 3,6,24,48
$GM = (3 \times 6 \times 24 \times 48)^{1/4}$
$= (3 \times 2 \times 3 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3)^{1/4}$
$= (2^4 \times 2^4 \times 3^4)^{1/4}$ [taking a group of four. four 2s and four 3s]
$= 2 \times 2 \times 3 = 12$

NOTE: whatever number we have in the power we have to make groups of that. Eg in Q2 we have 3 in the power that is given by (1/3) so we take or arrange the variables in a group of 3, eg (2x2x2). Similarly in Q3 we take a group of 4.

Sometimes the question is given in such a way that the value of GM is given and one of the variables is missing.

Q4) The GM of a, 4, 8 is 6. Find a.

$GM = (a \times 4 \times 8)^{1/3}$

$6 = (a \times 4 \times 8)^{1/3}$

Multiplying with cube on both sides, we get

$6^3 = [(a \times 4 \times 8)^{1/3}]^3$

⇨ $6 \times 6 \times 6 = a \times 4 \times 8$

⇨ $\frac{6 \times 6 \times 6}{4 \times 8} = a$

⇨ $a = 27/4$

⇨ $a = 6.75$

Solve the following questions.

1) Find the GM of
   1. 1, 3, 9. (ans= 3)
   2. 12, 3, 48. (ans= 12)
   3. 5, 4, 25, 20, 10. (ans= 10)

2) If the GM of z, 9, 12 is 6. Find the value of z. (ans= 2)

HARMONIC MEAN

$$HM = \frac{n}{\frac{1}{x1} + \frac{1}{x2} + \frac{1}{x3} + \cdots \ldots \frac{1}{xn}}$$

Q1) Find the HM of 3, 6, 24, 48

$$HM = \frac{4}{\frac{1}{3} + \frac{1}{6} + \frac{1}{24} + \frac{1}{48}}$$

Taking the LCM of the denominator

$$= \frac{4}{\frac{16+8+2+1}{48}}$$

$$= \frac{4}{\frac{27}{48}} \quad \Rightarrow 4 \times \frac{48}{27} \quad \Rightarrow \quad 7.11$$

HM = 7.11

Q2) A man travels from Delhi to Agra by car in 4 hours . in the first hour he travels at a speed of 50 km/hr , in the second hour he travels at a speed of 65km/hr, in the third hour he travels at a speed of 80km/hr and in the fourth hour he travels at a speed of 55 km/hr. Find the average speed at which he travels.

Ans

He travels at four different speeds 50, 65, 80 and 55

$$HM = \frac{4}{\frac{1}{50}+\frac{1}{65}+\frac{1}{80}+\frac{1}{55}}$$

$$=. \frac{4}{.020 + .015 + .013 + .018}$$

$$= \frac{4}{.066} \quad = 60.5$$

HM = 60.5
The average speed is 60.5 km/hr

NOTE: In this example we did not take the LCM of the denominator because the numbers are big as compared to Q1. When the numbers are big the sum can be ;solve this way too. Here we are dividing it individually eg, 1/50 is equal to .020. therefore you can use any of the two methods to solve it.

Solve.
Find the HM of

1) 4, 28, 2, 56, 14 (ans = 5.71)
2) 90, 45, 24 (ans 42.86)
3) A girl travels from Darjeeling to Siliguri by a taxi in 3hours . in the first hour the taxi travels at a speed of 50 km/hr , in the second hour the taxi travels at a speed of 45km/hr and in the third hour the taxi travels at a speed of 70km/hr. Find the average speed of the taxi (ans 60 km/hr)

RELATION BETWEEN AM, GM AND HM.

AM > GM > HM.

The relationship is given by the equation $AM \times HM = GM^2$

Q1) AM is given as 10 and HM is given as 3.6. Find the GM.
Ans

$AM \times HM = GM^2$

- ⇨ $10 \times 3.6 = GM^2$
- ⇨ $36 = GM^2$
- ⇨ Taking square root on both the sides
- ⇨ $(36)^{1/2} = GM$
- ⇨ $GM = 6$

Solve.
If the GM is given as 9 and the AM is 11. Find HM. (ans 7.36)